# BUKU PETUNJUK PRAKTIKUM 

## OPTIK



## JURUSAN FISIKA

FAKULTAS SAINS DAN TEKNOLOGI

## JUDUL PERCOBAAN

| No. | JUDUL PERCOBAAN |
| :---: | :--- |
| $\mathbf{1}$ | MICHELSON INTERFEROMETER |
| $\mathbf{2}$ | MAGNETOSTRICTION WITH THE MICHELSON INTERFEROMETER |
| $\mathbf{3}$ | POLARISATION THROUGH $\boldsymbol{\lambda} / 4$ PLATES |
| 4 | DIFFRACTION INTENSITY OF MULTIPLE SLITS AND GRIDS |

## Related topics

Interference, wavelength, refractive index, velocity of light, phase, virtual light source.

## Principle

In the Michelson arrangement interference will occur by the use of 2 mirrors. The wavelength is determined by displacing one mirror using the micrometer screw.

## Equipment

Michelson interferometer Laser, He-Ne $1.0 \mathrm{~mW}, 220$ V AC
Swinging arm
Lens, mounted, $f=+20 \mathrm{~mm}$
Lens holder
Slide mount f. opt. pr.-bench, $h=30 \mathrm{~mm}$
Optical profile bench $l=60 \mathrm{~cm}$
Base f. opt. profile-bench, adjust.
Screen, metal, $300 \times 300 \mathrm{~mm}$
Barrel base -PASS-
$08557.00 \quad 1$
08181.931
$08256.00 \quad 1$
$08018.01 \quad 1$
$08012.00 \quad 1$
08286.013
$08283.00 \quad 1$
$08284.00 \quad 2$
$08062.00 \quad 1$
02006.551

## Tasks

Determination of the wavelength of the light of the used laser.

## Set-up and procedure

The experimental set up is as shown in Fig. 1. In order to obtain the largest possible number of interference fringes, the two mirrors of the interferometer are first of all adjusted; to do this, the lens is first of all removed. The laser beam strikes the half-silvered mirror at an angle of $45^{\circ}$ splitting the beam. The resulting two beams are reflected by the mirror and impinge on the screen. By means of the two adjusting screws fitted to one of the mirrors, both points of light are made to coincide. If the lens is placed in the light beam, the points of light are enlarged and the interference patterns are observed on the screen (bands, circles). By careful readjustment, an interference image of concentric circles will be obtained.

To measure the wavelength, the micrometer screw is turned to any initial position at which the centre of the circles is dark. The micrometer screws is now further turned in the same direction and the light-dark periods thus produced are counted. The distance travelled by the mirror must be read off on the micrometer screw and divided by ten (lever reduction $1: 10)$. Should the central point of the circles move outside the light spot area a readjustment has to be performed.

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up for measuring wavelengths with the Michelson interferometer.


Fig. 2: Michelson interferometer set up.


## Theory and evaluation

If 2 waves of the same frequency $\omega$ but of different amplitude and different phase impinge on one point they are superimposed, or interfere, so that:

$$
y=a_{1} \sin \left(\omega t-\alpha_{1}\right)+a_{2} \sin \left(\omega t-\alpha_{2}\right) .
$$

The resulting wave can be described as

$$
y=A \sin (\omega t-\alpha)
$$

with the amplitude

$$
\begin{equation*}
A^{2}=a_{1}^{2}+a_{2}^{2}+2 a_{1} a_{2} \cos \delta \tag{1}
\end{equation*}
$$

and the phase difference

$$
\delta=\alpha_{1}-\alpha_{2} .
$$

In a Michelson interferometer, light is split up into two beams by a half-silvered glass plate (amplitude splitting), reflected by two mirrors, and passed again through the glass plate to produce interference phenomena behind it.

A lens is inserted between the light beam and the glass plate so that the light source lies at the focal point, since only enlarged light spots can exhibit interference rings.

If the actual mirror $\mathbf{M}_{1}$ is replaced by its virtual image $\mathbf{M}_{\mathbf{1}}^{\prime}=\mathbf{M}_{\mathbf{2}}$ which is formed by reflection at the glass plate, a point $\mathbf{P}$ of the real light source is formed as the points $\mathbf{P}^{\prime}$ and $\mathbf{P}$ " of the virtual light sources $\mathbf{L}_{1}$ and $\mathbf{L}_{\mathbf{2}}$.

Fig. 3: Formation of circles on interference.


Based on the different light paths, the phase difference, using the symbols of Fig. 3, is:

$$
\begin{equation*}
\delta=\frac{2 \pi}{\lambda} 2 d \cos \theta \tag{2}
\end{equation*}
$$

where $\lambda$ is the wavelength of the light used in the experiment.
The intensity distribution for $a_{1}=a_{2}=a$ according to (1) is:

$$
\begin{equation*}
I \sim A^{2}=4 a^{2} \cos ^{2} \frac{\delta}{2} \tag{3}
\end{equation*}
$$

Maxima thus occur if $\delta$ is a multiple of $2 \pi$, i.e. from equation (2), if

$$
\begin{equation*}
2 d \cos \theta=m \lambda ; m=1,2, \ldots \tag{4}
\end{equation*}
$$

i.e. circles are produced for a fixed value of $m$ and $d$ since $\theta$ remains constant (see Fig. 3).

If the position of the movable mirror $\mathbf{M}_{1}$ is changed so that $d$ for example decreases then, according to equation (4), the diameter of the ring will also decrease since $m$ is fixed for this ring. A ring thus dissapears each time $d$ is reduced by $\lambda / 2$. The ring pattern disappears if $d=0$.

If $\mathbf{M}_{\mathbf{1}}$ and $\mathbf{M}_{\mathbf{2}}$ are not parallel, curved bands are obtained which are converted to straight bands when $d=0$.

To measure the wavelength of the light, 500 ring changes were counted. A $158 \mu \mathrm{~m}$ displacement of the mirrow was measured. From this, the wavelength was obtained as:

$$
\lambda=632 \mathrm{~nm} .
$$

## Related terms

Interference, wavelength, diffraction index, speed of light, phase, virtual light source, ferromagnetic material, Weiss molecular magnetic fields, spin-orbit coupling.

## Principle

With the aid of two mirrors in a Michelson arrangement, light is brought to interference. Due to the magnetostrictive effect, one of the mirrors is shifted by variation in the magnetic field applied to a sample, and the change in the interference pattern is observed.

## Equipment

Base plate with rubber feet
08700.00

HeNe Laser, $1 \mathrm{~mW}^{*}$
Adjusting support, $35 \times 35 \mathrm{~mm}$
Surface mirror, $30 \times 30 \mathrm{~mm}$
08180.93

1

Magnetic base
08711.003
08711.014
08710.007
$08719.00 \quad 1$
Plate holder
$08741.00 \quad 1$
Lens with mount, $f=+20 \mathrm{~mm}$ 08018.01

Lens holder for base plate
$08723.00 \quad 1$
Screen, white, $150 \times 150 \mathrm{~mm}$
09826.00

Coil, $N=1200,4 \Omega$, Faraday Modulator
08733.00

Metal rods for magnetostriction
08733.01

Power supply, universal
13500.93

Digital multimeter
$07134.00 \quad 1$
Battery, 9 V, 6 F22
$07496.10 \quad 1$
Connecting cord, $l=500 \mathrm{~mm}$, blue
*alternative:
HeNe Laser, $5 \mathrm{~mW} \quad 08701.00$
Power supply and switch for 5 mW Laser

## Tasks

- Construction of a Michelson interferometer using separate optical components.
- Testing various ferromagnetic materials (iron and nickel) as well as a non-ferromagnetic material (copper) with regard to their magnetostrictive properties.


## Set-up and procedure

In the following, the pairs of numbers in brackets refer to the co-ordinates on the optical base plate in accordance with Fig. 1. These co-ordinates are only intended to be a rough guideline for initial adjustment.

- Perform the experimental set-up according to Fig. 1 and 1a. The recommended set-up height (beam path height) is 130 mm .
- The lens $\mathbf{L}[1,7]$ must not be in position when making the initial adjustments.
- When adjusting the beam path with the adjustable mirrors $\mathbf{M}_{\mathbf{1}}[1,8]$ and $\mathbf{M}_{\mathbf{2}}[1,4]$, the beam is set along the 4th $y$ coordinate of the base plate.
- Place mirror $\mathbf{M}_{3}$ onto the appropriate end of a sample (nickel or iron rod) - initially without the beam splitter BS [6,4] and screw it into place.

Fig. 1. Experimental set-up



- Now, insert the sample into the coil in such a manner that approximately the same length extends beyond the coil on both ends so that a uniform magnetisation can be assumed for the measurement. Fix the sample in position with the laterally attached knurled screw.
- Next, insert the coil C's shaft into a magnetic base and place it at position $[11,4]$ such that the mirror's plane is perpendicular to the propagation direction of the laser's beam (see Fig. 1a).
- Adjust the beam in a manner such that the beam reflected by mirror $\mathbf{M}_{3}$ once again coincides with its point of origin on mirror $\mathbf{M}_{\mathbf{2}}$. This can be achieved by coarse shifting of the complete unit of coil with magnetic base or by turning the sample rod with mirror $\mathbf{M}_{3}$ in the coil and by meticulously aligning mirror $\mathbf{M}_{\mathbf{2}}[1,4]$ with the aid of its fine adjustment mechanism.
- Next, position the beam splitter BS [6, 4] in such a manner that one partial beam still reaches mirror $\mathbf{M}_{3}$ without hindrance and the other partial beam strikes mirror $\mathbf{M}_{4}[6,1]$. Die metallized side of $\mathbf{B S}$ is facing mirror $\mathbf{M}_{4}$.
- Two luminous spots now appear on the screen SC [6, 6]. Make them coincide by adjusting the mirror $\mathbf{M}_{\mathbf{4}}$ until a slight flickering of the luminous spot can be seen.
- After positioning lens L [1,7], an illuminated area with interference patterns appears on the screen. To obtain concentric circles, meticulously readjust mirror $\mathbf{M}_{4}$ using the adjustment screws.
- Subsequent to the connection of the coil to the power supply (connect the multimeter in series between the coil and the power supply to measure the current, measuring range 10 AC!), set the DC-voltage to maximum and DC-current to minimum value. Then slowly readjust the current. For the measurements the resulting currents lie between 0.5 and, maximally, 5 A . Count the changes from maximum to maximum (or minimum to minimum) in the interference pattern. In addition, pay attention to the direction in which the circular interference fringes move (sources or sinks!).
Repeat this procedure using different samples and different current strengths $I$ between 0.5 and 5.0 A ( $I>3$ A only for a short time!).
- Notes:

The materials require a certain amount of premagnetisation; therefore, the current should be run up and down several times for each individual determination before performing the intensity change measurement.
The blank trial with a copper rod as sample should serve to demonstrate that the longitudinal deformation effect is due to magnetostriction and not to other causes.

## Theory and evaluation

If two waves having the same frequency $\omega$ but different amplitudes and different phases are coincident at one location, they superimpose to

$$
Y=a_{1} \cdot \sin \left(\omega t-\alpha_{1}\right)+a_{2} \cdot \sin \left(\omega t-\alpha_{2}\right) .
$$

Fig. 1a. Experimental set-up of the Michelson interferometer for the measurement of magnetostriction of different ferromagnetic materials (*only necessary with the 5-mW laser)


The resulting wave can be described by the following:

$$
Y=A \cdot \sin (\omega t-\alpha)
$$

with the amplitude

$$
\begin{equation*}
A^{2}=a_{1}^{2}+a_{2}^{2}+2 a_{1} a_{2} \cdot \cos \delta \tag{1}
\end{equation*}
$$

and the phase difference

$$
\delta=\alpha_{1}-\alpha_{2} .
$$

In a Michelson interferometer, the light beam is split by a halfsilvered glass plate into two partial beams (amplitude splitting), reflected by two mirrors, and again brought to interference behind the glass plate (Fig. 2). Since only large luminous spots can exhibit circular interference fringes, the light beam is expanded between the laser and the glass plate by a lens $\mathbf{L}$. If one replaces the real mirror $\mathbf{M}_{\mathbf{4}}$ with its virtual image $\mathbf{M}_{4}{ }^{\prime}$, which is formed by reflection by the glass plate, a point $\mathbf{P}$ of the real light source appears as the points $\mathbf{P}^{\prime}$ and $\mathbf{P \prime \prime}$ of the virtual light sources $L_{1}$ and $L_{2}$.
As a consequence of the different light paths traversed, and using the designations in Fig. 3, the phase difference is given by:

$$
\begin{equation*}
\delta=\frac{2 \pi}{\lambda} \cdot 2 \cdot d \cdot \cos \theta \tag{2}
\end{equation*}
$$

$\lambda$ is the wavelength of the laser light used.
According to (1), the intensity distribution for $a_{1}=a_{2}=a$ is:

$$
\begin{equation*}
I \sim A^{2}=4 \cdot a^{2} \cdot \cos ^{2} \frac{\delta}{2} \tag{3}
\end{equation*}
$$

Fig. 2. Michelson arrangement for Interference. $\mathbf{S}$ represents the light source; SC the detector (or the position of the screen)


Maxima thus occur when $\delta$ is equal to a multiple of $2 \pi$, hence with (2)

$$
\begin{equation*}
2 \cdot d \cdot \cos \theta=m \cdot \lambda ; m=1,2, \ldots \tag{4}
\end{equation*}
$$

i.e. there are circular fringes for selected, fixed values of $m$, and $d$, since $\Theta$ remains constant (see Fig.3).
If one alters the position of the movable mirror $\mathbf{M}_{\mathbf{3}}$ (cf. Fig.1) such that $d$, e.g., decreases, according to (4), the circular fringe diameter would also diminish since $m$ is indeed defined for this ring. Thus, a ring disappears each time $d$ is reduced by $\lambda / 2$. For $d=0$ the circular fringe pattern disappears. If the surfaces of mirrors $\mathbf{M}_{4}$ and $\mathbf{M}_{\mathbf{3}}$ are not parallel in the sense of Fig. 3, one obtains curved fringes, which gradually change into straight fringes at $d=0$.

Fig. 3. Formation of circular interference fringes


Fig.4: Magnetostriction of different ferromagnetic materials with their relative change in length $\Delta l / l$ plotted against applied magnetic field strength $H_{\mathrm{m}}$


In this experiment only the longitudinal magnetostriction is considered (see Fig. 4). One must take into consideration that the magnetostriction is a function of temperature and that premagnetisation is necessary. Additionally, the magnetostriction in alloys is also dependent on the composition of the metals and the appropriate pre-treatment (see Fig. 5).

## Thermodynamic description of magnetostriction:

Magnetostriction can be described quantitatively and thermodynamically using:

$$
\begin{array}{ll}
S: & \text { elastic tension } \\
s: & \text { elastic deformation (i.e.: } \Delta l / l) \\
B: & \text { magnetic induction } \\
H: & \text { Magnetic field strength } \\
\mu: & \text { magnetic permeability with } \\
& \frac{1}{\mu}=\frac{\partial H}{\partial B}
\end{array}
$$

E: Elasticity module with

$$
E=\frac{\partial S}{\partial s}
$$

As a result of thermodynamic relationships, it can be shown that the direct and reciprocal magnetostriction effects are mutually linked via

$$
\begin{equation*}
\frac{\partial S}{\partial B}=\frac{1}{4 \pi} \cdot \frac{\partial H}{\partial s} \tag{5}
\end{equation*}
$$

Fig. 5. Magnetostriction of different ferromagnetic alloys with their relative change in length $\Delta l / l$ plotted against applied field strength $H_{m}$


For a free rod (unloaded and not clamped in position), the following is true:

$$
\begin{equation*}
s=-\gamma \cdot \frac{B}{E} \tag{6}
\end{equation*}
$$

with the substance-specific quantity

$$
\gamma=\frac{\partial S}{\partial B} .
$$

In other words, the relative longitudinal change is given by

$$
\begin{equation*}
s=-\gamma \cdot \mu \cdot \frac{H}{E} \tag{7}
\end{equation*}
$$

In this context, $\gamma$ cannot be a constant as otherwise a linear increase in the relative length with the magnetic field strength would result. This is however not the case, since a saturation value is reached as of a specific field strength.

On the evaluation of the measuring results:
The magnetic field strength of a cylindrical coil is given by:

$$
\begin{equation*}
H_{\mathrm{m}}=\frac{N \cdot I}{\sqrt{4 \cdot r^{2}+l_{\mathrm{s}}^{2}}} \tag{8}
\end{equation*}
$$

where
$H_{\mathrm{m}}$ : magn. field strength at the centre of the coil in $A \cdot \mathrm{~m}^{-1}$
$r$ : $\quad$ Radius of a winding (here: 0.024 m )
$l_{\mathrm{s}}$ : Length of the coil (here: 0.06 m )
$N$ : Number of windings (here: 1200)

Fig. 6. Measuring results of the magnetostriction of iron (steel) with the relative change in length $\Delta l / l$ plotted against applied field strength $H$


On condition that the field is homogenous, the field strength is by the following for $l \gg r$ :

$$
\begin{equation*}
H=\frac{N \cdot I}{l} \tag{9}
\end{equation*}
$$

For this measurement we assume, as a first approximation, that the magnetic field strength $H_{\mathrm{m}}$ acts on the entire length of the $\operatorname{rod}(l=0.15 \mathrm{~m})$.
The alteration in length $\Delta l$ is obtained from the number of circular fringe changes $n$; in the process the separation per circular fringe change alters by $\lambda / 2(\lambda=632 \mathrm{~nm})$ :

$$
\begin{equation*}
\Delta l=n \cdot \lambda / 2 \tag{10}
\end{equation*}
$$

In Tables (1) and (2) the results of the measurements on nickel and iron are summarised. In the measurements, the direction of magnetostriction also became apparent:
In iron the radii of the interference rings increased with increasing magnetic field strength (sources!); thus, the rod must have become larger (see Fig. 6 and Table 1).

Table 1

| $/ \mathrm{A}$ | $H / \mathrm{A} / \mathrm{m}$ <br> $($ see (9)) | Ring- <br> changes $/ n$ | $\Delta l / \mathrm{m}$ <br> $($ see $(10))$ | $\Delta l / l$ <br> with $l=0.15 \mathrm{~m}$ |
| :--- | :--- | :--- | :---: | :---: |
| 0.83 | 16600 | $\approx 11 / 4$ | $0.395 \cdot 10^{-6}$ | $2.63 \cdot 10^{-6}$ |
| 1.27 | 25400 | $\approx 13 / 4$ | $0.554 \cdot 10^{-6}$ | $3.691 \cdot 10^{-6}$ |
| 1.6 | 32000 | $\approx 11 / 2$ | $0.475 \cdot 10^{-6}$ | $3.164 \cdot 10^{-6}$ |
| 1.87 | 37400 | $\approx 11 / 4$ | $0.395 \cdot 10^{-6}$ | $2.630 \cdot 10^{-6}$ |

Fig. 7. Measuring results of the magnetostriction of nickel with the relative change in length $\Delta l / l$ plotted against applied field strength $H$


In nickel the rod became shorter (sink of circular interference fringes); therefore, a negative magnetostriction existed in this case (see Fig. 7 and Table 2).

## Table 2

| $I / \mathrm{A}$ | $H / \mathrm{A} / \mathrm{m}$ <br> $($ see (9)) | Ring- <br> changes $/ n$ | $\Delta l / \mathrm{m}$ <br> $($ see (10)) $)$ | $\Delta l / l$ <br> with $l=0.15 \mathrm{~m}$ |
| :--- | :--- | :---: | :---: | :---: |
| 0.53 | 10600 | 4 | $-1.27 \cdot 10^{-6}$ | $-8.44 \cdot 10^{-6}$ |
| 0.71 | 14200 | 5 | $-1.58 \cdot 10^{-6}$ | $-10.55 \cdot 10^{-6}$ |
| 0.94 | 18800 | 6 | $-1.90 \cdot 10^{-6}$ | $-12.67 \cdot 10^{-6}$ |
| 1.33 | 26600 | 7 | $-2.21 \cdot 10^{-6}$ | $-14.77 \cdot 10^{-6}$ |
| 2.04 | 40800 | 8 | $-2.53 \cdot 10^{-6}$ | $-16.87 \cdot 10^{-6}$ |
| 3.28 | 65600 | 9 | $-2.84 \cdot 10^{-6}$ | $-18.98 \cdot 10^{-6}$ |

The comparison with the literature values (Fig. 4) exhibited good agreement. In copper no alteration in length can be detected for a rapid current elevation. It may be possible that a slow heating of the material would show changes in the circular interference rings over a long period of time.

## Related topics

Huyghens principle, interference, Fraunhofer- und Fresnel-diffraction, coherence, laser.

## Principle

Multiple slits which all have the same width and the same distance among each other, as well as transmission grids with different grid constants, are submitted to laser light. The corresponding diffraction patterns are measured according to their position and intensity, by means of a photo diode which can be shifted.

## Equipment

Laser, He-Ne 1.0 mW , 220 V AC Universal measuring amplifier Optical profile bench $l=150 \mathrm{~cm}$ Base f. opt. profile-bench, adjust.
08181.93 $13626.93-1$ Slide mount f. opt. pr.-bench, $h=30 \mathrm{~mm}$ Slide device, horizontal Lens holder
Object holder, $5 \times 5 \mathrm{~cm}$ Lens, mounted, $f=+20 \mathrm{~mm}$
Lens, mounted, $f=+100 \mathrm{~mm}$
Photoelement f. opt. base plt.
Diaphragm, 3 single slits
$08281.00 \quad 1$
$08284.00 \quad 2$
$08286.01 \quad 5$
$08713.00 \quad 1$
$08012.00 \quad 2$
$08041.00 \quad 1$
$08018.01 \quad 1$
$08021.01 \quad 1$
$08734.00 \quad 1$
$08522.00 \quad 1$
Diaphragm, 4 multiple slits
Diffraction grating, 4 lines $/ \mathrm{mm}$
$08526.00 \quad 1$
Diffraction grating, 8 lines $/ \mathrm{mm}$
Diffraction grating, 10 lines $/ \mathrm{mm}$

Diffraction grating, 50 lines $/ \mathrm{mm}$
08543.00

1
Digital multimeter
07122.00

Connecting cord, $l=750 \mathrm{~mm}$, red
Connecting cord, $l=750 \mathrm{~mm}$, blue
07362.01
$07362.04 \quad 1$

## Tasks

1. The position of the first intensity minimum due to a single slit is determined, and the value is used to calculate the width of the slit.
2. The intensity distribution of the diffraction patterns of a threefold, fourfold and even a fivefold slit, where the slits all have the same widths and the same distance among each other, is to be determined. The intensity relations of the central peaks are to be assessed.
3. For transmission grids with different lattice constants, the position of the peaks of several orders of diffraction is to be determined, and the found value used to calculate the wavelength of the laser light.

## Set-up and procedure

Experimental set-up is shown in fig. 1. With the assistance of the $f=20 \mathrm{~mm}$ and $f=100 \mathrm{~mm}$ lenses, a widened and parallel laser beam is generated, which must impinge centrally on the photocell with the slit aperture, the photocell being situated approximately at the centre of its shifting range. The diffract$08532.00 \quad 1 \quad$ ing objects are set in the object holder. It must be made sure $08534.00 \quad 1$ the diffraction objects which are to be investigated are set ver08540.001 tically in the object holder, and uniformly illuminated.

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up to investigate the diffraction intensity of multiple slits and grids. (Positions of the components on the optical bench: laser $=2.5 \mathrm{~cm} ; f / 20 \mathrm{~mm}$ lens $=14.5 \mathrm{~cm} ; f / 100 \mathrm{~mm}$ lens $=27.5 \mathrm{~mm}$; diffracting objects $=33 \mathrm{~cm}$; slide mount lateral adjustm., calibr. $=147.5 \mathrm{~cm}$ ).


## LEP

2.3.04

Diffraction intensity of multiple slits and grids

The laser and the measuring amplifier should warm up for about 15 minutes before starting measurements, in order to avoid undesirable intensity fluctuations. The photocell is connected to the $10^{4} \Omega$ input of the measuring amplifier (amplification factor $10^{3}-10^{5}$ ). When the amplification factor is changed, the zero point of the measurement amplifier must be checked while the photocell is covered, and corrected if necessary.
The diffraction intensity values are determined for the multiple slits by shifting the photocell in steps of $0.1 \mathrm{~mm}-0.2 \mathrm{~mm}$. For the transmission grids, the positions of diffraction peaks must be determined so as to be able to calculate the wavelength of the laser light. For the 50 lines/mm transmission grid, the secondary peaks are outside the shifting range of the photocell, so that in this case the position of the diffraction reflexes must be marked on a sheet of paper and their distance measured with a ruler.

## Theory and evaluation

If monochromatic light of wavelength $\lambda$ impinges on a system of parallel and equidistant slits, the following will be true for the light intensity $I$ of beams deflected by an angle $\varphi$ :

$$
\begin{equation*}
I(\varphi) \propto b^{2} \cdot \frac{\sin ^{2}\left(\frac{\pi}{\lambda} \cdot b \sin \varphi\right)}{\left(\frac{\pi}{\lambda} \cdot b \sin \varphi\right)^{2}} \cdot \frac{\sin ^{2}\left(\frac{p \pi}{\lambda} \cdot g \sin \varphi\right)}{\sin ^{2}\left(\frac{\pi}{\lambda} \cdot g \sin \varphi\right)} \tag{1}
\end{equation*}
$$

( $b=$ width of slit; $g=$ distance between slits; $p=$ number of slits)


Fig. 2: Diffraction intensity $I$ as a function of the position $x$ for a threefold slit, $b_{1}=0.1 \mathrm{~mm}$ and $g=0.25 \mathrm{~mm}$. Distance between threefold slit and photocell: $L=107 \mathrm{~cm}$. For comparison, the intensity distribution of a single slit, $b=0.1 \mathrm{~mm}$, is entered as a dotted line.

According to Fraunhofer, the minima and the peaks of a single slit are called $1^{\text {st }}$ class interferences, whereas the interaction of several slits yields $2^{\text {nd }}$ class interferences.
Observing only a single slit ( $1^{\text {st }}$ factor), this yields a minimum intensity when the numerator becomes zero. In this case, the following is valid:

$$
\begin{equation*}
\sin \varphi_{\mathrm{k}}=\frac{k \cdot \lambda}{b} ;(k=1,2,3 \ldots) \tag{2}
\end{equation*}
$$

The angular position of the $1^{\text {st }}$ class peaks is given approximately through:

$$
\begin{equation*}
\sin \varphi_{k}^{*}=\frac{2 k^{*}+1}{2} \cdot \frac{\lambda}{b} ;\left(k^{*}=1,2,3 \ldots\right) \tag{3}
\end{equation*}
$$

If several slits act together, the minima of the single slits always remain. Supplementary $2^{\text {nd }}$ class minima appear when the $2^{\text {nd }}$ factor also becomes zero.
For a double slit ( $p=2$ ), the zero points can be easily calculated by simple transformation of the $2^{\text {nd }}$ factor. Equation (1) then yields:

$$
\begin{equation*}
4 \cos ^{2}\left(\frac{\pi}{\lambda} \cdot g \sin \varphi\right)= \pm 1 \tag{4}
\end{equation*}
$$

This expression becomes zero for

$$
\begin{equation*}
\sin \varphi_{\mathrm{h}}=\frac{2 \mathrm{~h}+1}{2} \cdot \frac{\lambda}{g} ;(h=0,1,2,3 \ldots) \tag{5}
\end{equation*}
$$



Fig. 3: Diffraction intensity $I$ as a function of the position $x$ for a fourfold slit with $b_{1}=0.1 \mathrm{~mm}$ and $g=0.25 \mathrm{~mm}$.

Fig. 4: Reciprocal distance of the diffraction peaks up to the $3^{\text {rd }}$ order of diffraction $(K=3)$ as a function of the lattice constant.


The following is valid for the intensity $I$ of the main $2^{\text {nd }}$ class peaks:

$$
\begin{equation*}
I \propto p^{2} \tag{6}
\end{equation*}
$$

The main $2^{\text {nd }}$ class peaks thus become more prominent as the number of slits increases. There still are $(p-2)$ secondary $2^{\text {nd }}$ class peaks between the main peaks.
When light is diffracted through transmission grids with lattice constant g , the diffraction angle $\varphi$ of the main peaks fulfils the following relation:

$$
\begin{equation*}
\sin \varphi_{\mathrm{k}}=\frac{k \lambda}{g} ;(k=0,1,2,3 \ldots) \tag{7}
\end{equation*}
$$

Fig. 2 shows the diffraction intensity $I$ for a threefold slit as a function of the position $x$ of the photocell (distance between the diffracting object and the photocell; $L=107 \mathrm{~cm}$ ). For comparison, the diffraction pattern of a single slit is entered as an envelope, with an adapted ordinate scale.
The minima of the single slit also remain in presence of corresponding multiple slits. For these, one obtains $d=0.095 \mathrm{~mm}$ from (2), with the distance $2 \cdot \Delta x=14 \mathrm{~mm}$ between the two $1^{\text {st }}$ class minima $(\sin \varphi \approx \tan \varphi, L=107 \mathrm{~cm}, \lambda=632.8 \mathrm{~nm}$ ). The number of secondary $2^{\text {nd }}$ class peaks of the threefold slit is $(p-2)=1$.

Fig. 3 shows the diffraction figure of a fourfold slit. In this case, the number of $2^{\text {nd }}$ class peaks is $(p-2)=2$. In the same way, diffraction through a fivefold slit (no figure) yields $(p-2)=5-2^{\text {nd }}$ class secondary peaks.
Table 1 gives the intensity values of the central peaks of the diffracting objects with $p=3$ till $p=5$, as well as the relative values determined empirically and according to (6).

Table 1

|  | exp. | theor. |
| :--- | :--- | :--- |
| $I_{05}(p=5)=720$ Skt. |  |  |
| $I_{04}(p=4)=500$ Skt. | $I_{05} / I_{04}=1.44$ | $(5 / 4)^{2}=1.56$ |
| $I_{03}(p=3)=300$ Skt. | $I_{05} / I_{03}=2.40$ | $(5 / 3)^{2}=2.78$ |

Fig. 4 shows the distances between diffraction peaks measured for 4 different transmitting grids up to the $3^{\text {rd }}$ order ( $K=3$ ) as a function of the lattice constant g. With (7), fig. 4 yields $\lambda=635 \mathrm{~nm}$ as an average value for the wavelength of the used laser light.

## Polarisation through $\lambda / 4$ plates

## LP

3.2

## Used concepts

Linearly, circularly and elliptically polarised light, polarizer, analyser, plane of polarisation, Malus' law, double refraction, optical axis, ordinary and extraordinary beam.

## Principle

Monochromatic light impinges on a mica plate, perpendicularly to its optical axis. If the thickness of the plate is adequate ( $\lambda / 4$ plate), a phase shift of $90^{\circ}$ occurs between the ordinary and the extraordinary beam when the latter leaves the crystal. The polarisation of exiting light is examined for different angles between the optical axis of the $\lambda / 4$ plate and the direction of polarisation of incident light.

## Equipment

$\begin{array}{lll}\text { Base plate with rubber feet } & 08700.00 & 1 \\ \text { HeNe laser } & 08180.93 & 1 \\ \text { Adjusting support } 35 \times 35 \mathrm{~mm} & 08711.00 & 1 \\ \text { Surface mirror } 30 \times 30 \mathrm{~mm} & 08711.01 & 1 \\ \text { Magnet foot } & 08710.00 & 6 \\ \text { Lens support } & 08723.00 & 1 \\ \text { Mounted lens, } f=+20 \mathrm{~mm} & 08018.01 & 1 \\ \text { Aperture support } & 08724.00 & 1 \\ \text { Polarisation filter } & 08730.00 & 2 \\ \text { Mica polarising substance } & 08664.00 & 2 \\ \text { Photocell, silicone } & 08734.00 & 1 \\ \text { Measurement amplifier, universal } & 13626.93 & 1 \\ \text { Voltmeter } 0.3 \ldots 300 \mathrm{~V} / 10 \ldots 300 \mathrm{~V} \sim & 07035.00 & 1 \\ \text { Connecting cable, red, } I=500 \mathrm{~mm} & 07361.01 & 2\end{array}$

Fig. 1: Experimental set up for polarisation through $\lambda / 4$ plate (* only required for 5 mW laser)


## Problem

1. Measurement of the intensity of linearly polarised light as a function of the analyser's position (Malus' law: $I=I_{0} \cos ^{2} \phi$ ).
2. Measurement of light intensity after the analyser as a function of the angle between the optical axis of the $\lambda / 4$ plate and the analyser.
3. Carrying out experiment (2) with two successive $\lambda / 4$ plates.

## Set-up and performance

- The experimental set up is shown in fig. 1. The recommended set up height (height of beam path) should be 130 mm .
- To begin with, the beam path is adjusted without $\lambda / 4$ plate $\mathbf{G}$ in such a way that photocell LD is well illuminated. (Adjust amplification in so that voltage does not increase above the allowed maximum).
- The zero of the universal measurement amplifier is adjusted with the laser switched off.
- With polarizer $\mathbf{P}$ set to zero, analyser $\mathbf{A}$ is rotated until the intensity of transmitted light displays a minimum.
- The $\lambda / 4$ plate is clamped in the support and rotated until the light transmitted by the analyser again displays an intensity minimum.
- The direction of polarisation of light coming from the polarizer now forms an angle of $0^{\circ}$ (or $90^{\circ}$ ) with the optical axis of the $\lambda / 4$ plate.
- Light intensity is measured as a function of the position of the analyser within a range of $-90^{\circ}$ and $+90^{\circ}$, for the angles $0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$ and $90^{\circ}$ of the $\lambda / 4$ plate.
- Voltage supplied by the photocell is proportional to the intensity of incident light.


## Theory and evaluation

Light velocity in the direction of light polarisation has the same value $c_{0}$ as in the direction of the optical axis of a double refracting crystal. Polarised light also propagates with velocity $c_{0}$ perpendicularly to the optical axis, if the electric vector is perpendicular to the optical axis (ordinary beam, fig. 2). If the electric field vector is parallel to the optical axis, light velocity $c \neq c_{0}$ (extraordinary beam).
$E_{0}$ is the amplitude of an electric field vector coming out or polarizer $\mathbf{P}$ and $\phi$ the angle between polarisation direction $P$ and the optical axis of a double refracting crystal.

From fig. 2, one obtains the following relations for the field vectors of the ordinary and extraordinary beams:

$$
\begin{align*}
& E_{1}(t)=E_{0}(t) \cdot \sin \Phi  \tag{1a}\\
& E_{2}(t)=E_{0}(t) \cdot \cos \Phi \tag{1b}
\end{align*}
$$

At time $t$, the status of oscillation of the two beams at the surface of the crystal is described by:

$$
\begin{align*}
& E_{1}(t)=E_{0} \cdot \sin \Phi \cdot \sin \omega t  \tag{2a}\\
& E_{2}(t)=E_{0} \cdot \cos \Phi \cdot \sin \omega t \tag{2b}
\end{align*}
$$

For double refracting crystals ( $\lambda / 4$ plates) of thickness

$$
\begin{equation*}
d_{\lambda / 4}=\frac{\lambda}{4} \cdot \frac{1}{n_{0}-n_{\mathrm{a} 0}} \tag{3}
\end{equation*}
$$

( $n_{0}$ is the refraction index of the ordinary beam, $n_{\mathrm{a} 0}$ that of the extraordinary beam in the crystal)

Fig. 2: Splitting of polarised light in a double refracting crystal ( $\mathbf{P}=$ polarizer, $\mathbf{A}=$ analyser)

the two beams reunite to a resulting beam when leaving the crystal. This is accompanied by a path difference of $\lambda / 4$, that is, a phase difference of $\pi / 2$. Equation (2) becomes:

$$
\begin{align*}
& E_{x}=E_{1}=E_{0} \cdot \sin \Phi \cdot \sin \omega t  \tag{4a}\\
& E_{y}=E_{2}=E_{0} \cdot \sin \Phi \cdot \sin \omega t \tag{4b}
\end{align*}
$$

(4) is the parametric representation of an $E$ vector rotating around an axis in the direction of propagation. For $\phi=0^{\circ}$ and $\phi=90^{\circ}$, one obtains linearly polarised light with the intensity

$$
\begin{equation*}
J=J_{0} \sim E_{0}^{2} \tag{5}
\end{equation*}
$$

For $\phi=45^{\circ}$, one has

$$
\sin \Phi=\cos \Phi=\frac{1}{\sqrt{2}}
$$

and the absolute value of the rotating $E$ vector is:

$$
\begin{equation*}
E=\sqrt{E_{\mathrm{x}}^{2}+E_{\mathrm{y}}^{2}}=\frac{E_{0}}{\sqrt{2}} \tag{6}
\end{equation*}
$$

Fig. 3: Intensity distribution of linearly polarised light as a function of analyser position (without $\lambda / 4$ plate)


Light is circularly polarised, with intensity

$$
\begin{equation*}
J=\frac{J_{0}}{2} \sim \frac{E_{0}^{2}}{2} \tag{7}
\end{equation*}
$$

and is transmitted without being weakened for every position of analyser $\mathbf{A}$.
For all angles $\phi \neq 0^{\circ}, 45^{\circ}$, and $90^{\circ}$, transmitted light is polarised elliptically. The tip of an Evector rotating in the direction of propagation describes an ellipse with the half axes

$$
\begin{align*}
& E_{\mathrm{a}}=E_{0} \cdot \sin \Phi(\mathrm{x} \text {-direction })  \tag{8a}\\
& E_{\mathrm{b}}=E_{0} \cdot \cos \Phi(\mathrm{y} \text {-direction }) \tag{8b}
\end{align*}
$$

For light intensities transmitted by the analyser in the corresponding directions, the following relations hold:

$$
\begin{align*}
& J_{\mathrm{a}} \sim E_{\mathrm{a}}^{2}=E_{0}^{2} \cdot \sin ^{2} \Phi  \tag{9a}\\
& J_{\mathrm{b}} \sim E_{\mathrm{b}}^{2}=E_{0}^{2} \cdot \cos ^{2} \Phi \tag{9b}
\end{align*}
$$

Fig. 4: Intensity distribution of polarised light for different angles of the $\lambda / 4$ plate, as a function of the analyser position)


|  |
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| LP |
| 3.2 |

## Polarisation through $\lambda / 4$ plates

When the analyser is rotated, one obtains for the relation between maximum and minimum transmitted light intensity:

$$
\begin{equation*}
\frac{J_{\mathrm{a}}}{J_{\mathrm{b}}}=\frac{E_{\mathrm{a}}^{2}}{E_{\mathrm{b}}^{2}}=\frac{\sin ^{2} \Phi}{\cos ^{2} \Phi}=\tan ^{2} \Phi \tag{10}
\end{equation*}
$$

For an arbitrary angular position of the analyser as compared to the optical axis of the $\lambda / 4$ plates, the following is valid (Fig.2):

$$
\begin{equation*}
J \sim E_{0}^{2} \cdot \cos ^{2} \Phi \cdot \cos ^{2} \varphi+E_{0}^{2} \cdot \sin ^{2} \Phi \cdot \sin ^{2} \varphi \tag{11}
\end{equation*}
$$

To start with, the intensity distribution of linearly polarised light is measured without $\lambda / 4$ plate in the beam path as a function of analyser position (fig. 3). The form of polarisation of the outgoing light is determined from the corresponding intensity distributions for different angles between the optical axis of the $\lambda / 4$ plate and the transmitting direction of the analyser.
In case of two successive $\lambda / 4$ plates, linearly polarise light is generated for all angles
between the optical axis of the thus formed $\lambda / 2$ plate (fig. 5).


Fig. 5: Intensity distribution of polarised light for different angular positions of a $\lambda / 2$ plate

